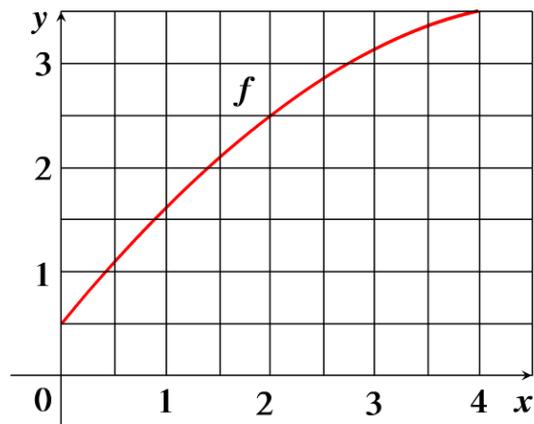
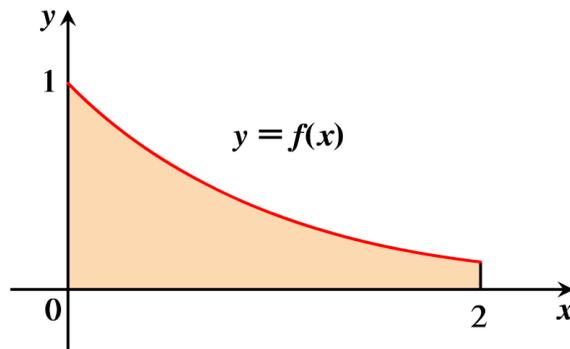


In the following problems  $L_n$  represents a left endpoint approximation,  $R_n$  represents a right endpoint approximation,  $M_n$  represents a midpoint approximation,  $T_n$  represents a trapezoidal approximation, and  $S_n$  represents a Simpson's approximation where  $n$  is the number of subintervals.

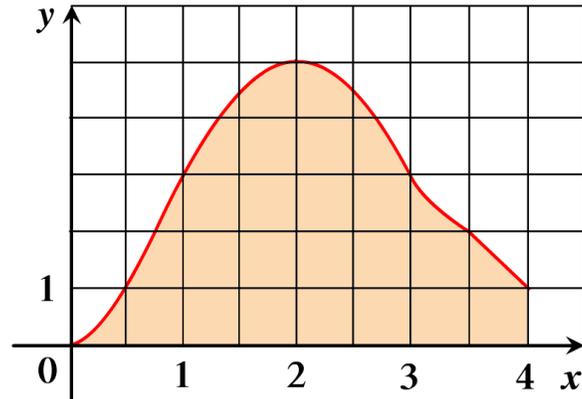
- 1) Let  $I = \int_0^4 f(x) dx$ , where  $f$  is the function whose graph is shown.
- Use the graph to find  $L_2$ ,  $R_2$ , and  $M_2$ .
  - Are these underestimates or overestimates of  $I$ ?
  - Use the graph to find  $T_2$ . How does it compare with  $I$ ?
  - For any value of  $n$ , list the numbers  $L_n$ ,  $R_n$ ,  $M_n$ ,  $T_n$ , and  $I$  in increasing order.



- 2) The left, right, Trapezoidal, and Midpoint Rule approximations were used to estimate  $\int_0^2 f(x) dx$ , where  $f$  is the function whose graph is shown. The estimates were 0.7811, 0.8675, 0.8632, and 0.9540, and the same number of subintervals were used in each case.
- a) Which rule produced which estimate?
- b) Between which two approximations does the true value of  $\int_0^2 f(x) dx$  lie?



- 3) Estimate the area under the graph in the figure by using the Trapezoidal Rule, the Midpoint Rule, and Simpson's Rule, each with  $n = 4$ .



- 4) Estimate  $\int_0^1 \cos(x^2) dx$  using the Trapezoidal Rule and the Midpoint Rule, each with  $n = 4$ . From a graph of the integrand, decide whether your answers are underestimates or overestimates. Between which two approximations does the true value of  $\int_0^1 \cos(x^2) dx$  lie?

Use the Trapezoidal Rule, the Midpoint Rule, and Simpson's Rule to approximate the given integral with the specified value of  $n$ . (Round your answers to six decimal places.)

5)  $\int_0^{1/2} \sin(x^2) dx, n = 4$

6)  $\int_1^2 e^{1/x} dx, n = 4$

7)  $\int_0^3 \frac{1}{1+y^5} dy, n = 6$

8) For the integral  $\int_0^2 e^{-x^2} dx$  find the following:

- a) Approximations  $T_{10}$  and  $M_{10}$ .
- b) Estimate the errors in the approximations of part a).
- c) How large do we have to choose  $n$  so that the approximations  $T_n$  and  $M_n$  to the integral in part a) are accurate to within 0.00001?

9) Use the following data to answer the following:

| $x$ | $f(x)$ |
|-----|--------|
| 0.0 | 6.8    |
| 0.4 | 6.5    |
| 0.8 | 6.3    |
| 1.2 | 6.4    |
| 1.6 | 6.9    |
| 2.0 | 7.6    |
| 2.4 | 8.4    |
| 2.8 | 8.8    |
| 3.2 | 9.0    |

a) Use the Midpoint Rule to estimate the value of the integral  $\int_0^{3.2} f(x) dx$ .

b) If it is known that  $-4 \leq f''(x) \leq 1$  for all  $x$ , estimate the error involved in the approximation in part a).